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# Contourlet Based Image Registration Using Blur Invariants

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Abstract—Analysis and interpretation of an image acquired from a nonideal imaging system has been the key problem in many application areas. The observed images are basically corrupted by blur, spatial degradations, and random noise. Methods like blind deconvolution and deblurring methods try to estimate the blur parameters and to restore the image. In this paper, we have proposed an alternative approach of deriving the features for image representation which are invariant with respect to blur regardless of the degradation PSF known as blur invariants. We have used a two dimensional transform that can capture the intrinsic geometrical structure which is the key for visual information. The major challenge in exploring the geometry in images comes from the discrete nature of the data. Unlike other approaches, such as curvelets, in which the transform is developed in the continuous domain and then discretized as sampled data, our proposed system begins with a discrete-domain construction and then studies its convergence to an expansion in the continuous domain. This construction then results in a very flexible multiresolution, local, and directional image expansion using the contour segments, and thus it is named the Contourlet transform. In this paper a template image is chosen from the degraded images and using the similarity measure the template image is matched with the original image. Inspite of severe degradations the template images are accurately registered using the Contourlet based blur invariants. It is also proved that the proposed Contourlet Transform can significantly outperform the wavelets in terms of PSNR (by several DB's).

*Index Terms*—blur invariants, image registration, Contourlet transform and template matching.

# I. INTRODUCTION

Blurring caused due to object or camera motion during image capture lead to substantial degradation in image quality. A great deal of research is carried on developing methods for restoring motion and noise blurred images. These methods make assumptions on the blurring process, the ideal image, and the noise. Many image processing techniques have been used to identify the blur and restore the image. The commonly used general model for the observed signal is

Y[n] = Bx[n] + Z[n]

In this model, Y is the observed image; x and z are the actual image and noise, respectively; and B is the degradation operator. If it is assumed that B is linear and space invariant for 2-D discrete signals, the general model can be simplified to,

$$Y [n1, n2] = b*x [n1, n2]$$

where '\*' denotes the convolution, and is the point spreadfunction of the system. In practical applications image

consist of different degradations due to the state of distortions with blur, which can result from impression turbulence, misfocus or with reference to motion between camera and the picture. The relation between ideal image f(x, y) and an observed image g(x, y) is given by,

g(x,y)=f(x,y)\*h(x,y)+n(x,y)

where h(x, y) is the point spread function of the system, n(x,y) is noise and \* denotes 2D convolution spread .

In some cases, the PSF is known prior tothe restoration process or it can be easily estimated, for instance from a point source in the image. This restoration problem has been treated by a lot of techniques, the most popular of which are inverse and Wiener filtering [3] and constrained deconvolution methods [4], [5]. In most applications the blur is unknown. Partial informationabout the PSF or the true image is sometimes available. There are basically two different approaches to degraded image analysis in that case: blind restoration and direct analysis. Blind image restoration [6], [7] is a process of estimating both the original image and the PSF from the degraded image using partial information about the imaging system. However, this is an ill-posed problem, which does not have a unique solution and the computational complexity of which could be extremely high. Direct analysis of the degraded image is based on the different idea. In many cases, one does not need to know the whole original image, one only needs for instance to localize or recognize some objects on it (typical examples are matching of a template against a blurred aerial image or recognition of blurred characters). In such cases, only knowledge of some representation of the objects is sufficient. However, such a representation should be independent of the imaging system and should really describe the original image, not the degraded one. In other words, we are looking for a functional I which is invariant to the degradation operator, i.e., I(g) = I(f). Contourlet based blur invariant descriptor has been proposed in this paper. The main advantages of these invariants will involve the exploration of a directional extension of the multidimensional wavelet transform called, 'Contourlets'. The paper is organized in such a way that the first section deals with the blur invariants and their basic definitions. In the next section a detailed explanation about Contourlet transform and their application has been discussed. In the last section of this paper various experiments have been carried out to show the performance of the Contourlet transform.



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#### II. BLUR INVARAINTS

Functional, invariant to convolution with arbitrary Centro symmetric PSF in image analysis literature they are often called "blur invariants" because common PSF's have a character of a low-pass filter. The blur invariants were first discovered by Flusseret al[8]. These invariant descriptors were developed in the spatial domain based on geometric moments. They assumed that the blur operator is symmetric. Later, they developed a closed format of the invariants [9] and decided to add some extra properties to the descriptors to make them invariant to geometric distortions changing their assumption for the blur systems to centrally symmetric [10], [11]. They also used these invariants for 1-D signals [12]. Using complex moments, Flusser and Zitovà proposed descriptors that are both invariant to centrally symmetric blur and rotation [13]. Instead of using geometric moments, Zhang et al. [16] employed Legendre moments in order to define their invariants in the spatial domain. Flusser and Suk [12] proposed Fourier-based invariants for 1-D signals based on the tangent of the phase of signals and proved that they are invariant to blur. They developed this representation for 2-D signals and showed their relationship with their invariants in the spatial domain [8]. Ojansivuand Heikkilä [19], proved that Flusser's invariants are sensitive to noise when implemented in the Fourier domain because of their use of a tangent operator. They proposed a different representation of invariants in the Fourier domain. Subsequently, they made them invariant to the affine transform as well [20]. Blur moment invariants have been used in vast area of research like image registration [10], [21]by template matching, remote sensing [22] in angiography, [23], forgery detection [24], recognition of blurred images [9], [14], [25], stereo matching [26], and control point extraction [27].

#### A. Definitions

Some of the basic definitions are reviewed, complimentary ones are proposed.

Definition 1: Image is a real discrete function  $x \in L^1(\mathbb{Z}^2)$ 

Definition 2: The ordinary geometric moment of order (p+q)

Definition 2: The ordinary geometric moment of o of in the spatial domain is defined by 
$$m_{p,q}{}^x = \sum_{n1 \in N1} \sum_{n2 \in N2} n_1{}^p n_2{}^q x [n1, n2]$$
 Definition 3: The centroid of signal is 
$$c_1{}^x = \frac{m_{1,0}{}^x}{m_{0,0}{}^x}, c_2{}^x = \frac{m_{0,1}{}^x}{m_{0,0}{}^x}$$

$$c_1^x = \frac{m_{1,0}^x}{m_{0,0}^x}$$
,  $c_2^x = \frac{m_{0,1}^x}{m_{0,0}^x}$ 

Definition 4: The central moment of order (p+q) of in the

spatial domain is defined by
$$\mu_{p,q}^{x} = \sum_{n1 \in \mathbb{N}_{1}} \sum_{n2 \in \mathbb{N}_{2}} (n_{1} - c_{1})^{p} (n_{2} - c_{2})^{q} x[n1, n2]$$

Definition 5:Let functional  $C:L_1(\mathbb{R}^2) \times N_0 \times N_0 \to \mathbb{R}$  be defined as follows:

• If (p+q) is even then  $C(p,q)^{(f)} = 0$ 

$$C(p,q)^{(f)}=0$$

• If (p+q) is odd then

$$C(p,q)^{(f)} = \mu_{pq}^{(f)} - \frac{1}{\mu_{00}^{(f)}} \sum_{n=0}^{p} \sum_{\substack{m=0\\0 < n+m < p+q}}^{q} \binom{p}{n} \binom{q}{m} C(p-n,q-m)^{(f)} \mu_{nm}^{(f)}$$

$$C(p,q)^{(f*h)} = C(p,q)^{(f)}$$

#### III. CONTOURLET TRANSFORM

The contourlet transform is a discrete extension of the curvelet transform that aims to capture curves instead of points, and provides for directionality and anisotropy. The curvelet transform was first developed in the continuous domain [28] by multiscale filtering and then applying a block ridge let transform [30] on each band pass image. Later, they proposed a second generation curvelet transform [29]that were defined directly via frequency partitioning without using ridge let transform. Both curvelet constructions require a rotation operation and correspond to a 2-D frequency partitionbased on the polar coordinate. This makes the curvelet construction simple in the continuous domain but causes theimplementation for discrete images - sampled on a rectangulargrid - to be very challenging. In particular, approaching criticalsampling seems difficult in such discretized constructions. The reason for this difficulty, we believe, is because the typical rectangular-sampling grid imposes a prior geometry todiscrete images; e.g. strong bias toward horizontal and vertical directions. This fact motivates our development of a directional multiresolution transform like curvelets, but directly in the discrete domain, which results in the contourlet construction described in this paper. We would like to emphasize that although curvelet and contourlet transforms have some similar properties and goals, the latter is not a discretized version of the former. Apart from curvelets and contourlets, there have recentlybeen several approaches in developing efficient representations of geometrical regularity. Notable examples are band lets [31], the edge-adapted multiscale transform [32], wedge lets [33],[34], and quad tree coding [35]. These approaches typicallyrequire an edge-detection stage, followed by an adaptive representation. By contrast, curvelet and contourlet representations are fixed transforms. This feature allows them to be easilyapplied in a wide range of image processing tasks, similar towavelets. For example, we do not have to rely on edge detection, which is unreliable and noise sensitive. Furthermore, wecan benefit from the well-established knowledge in transform coding when applying contourlets to image compression (e.g.for bit allocation). Several other well-known systems that provide multiscale and directional image representations include: 2-D Gaborwavelets [36], the cortex transform [37], the steerable pyramid[38], 2-D directional wavelets [39], brushlets [40], and complex wavelets [41]. The main differences between these systems and our contourlet construction are that the previousmethods do not allow for a different number of directionsat each scale while achieving nearly critical sampling. Inaddition, our



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construction employs iterated filter banks, whichmakes it computationally efficient, and there is a preciseconnection with continuous-domain expansions.

ULL 1

HL 2

WT LH 4

HH 4

HH 5

Mayelet Decomposition Directional Filter Bank

Fig. 1-Level Contourlet decomposition

Figure 1 the Laplacianpyramid (LP), is first used to capture the point discontinuities followed by a directional filter bank (DFB) to link point discontinuities into linear structures. The LP decomposition at each level generates a down sampled low pass version of the original data. The process can be iterated on the down sampled signal. The new DFB-based quincunx filter bank (QFB) which avoids the modulation of the input images and has a simpler rule for expanding the decomposition tree is adopted by contourlet transform. The DFB is designed to capture high frequency components of images.

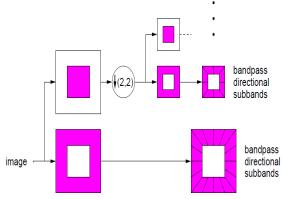


Fig-2The Contour let filter bank

Figure 2 shows a multiscale and directional decomposition using a combination of a Laplacian pyramid (LP) and a directional filter bank (DFB). Band pass images from the LP are fed into a DFB so that directional information can be captured. The scheme can be iterated on the coarse image. The combined result is a double iterated filter bank structure, named contourlet filter bank, which decomposes images into directional sub bands at multiple scales. Figure 3 shows examples of the contourlet transform. We notice that only contourlets that match with both location and direction of image contours produce significant coefficients. For clear visualization; each image is only decomposed into two pyramidal levels, which are then decomposed into four and

eight directional sub bands. Small coefficients are shown in black while large coefficients are shown in white.

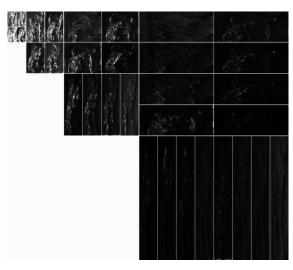


Fig-3 Example of the Contourlet transform on Lenaimage

#### IV. EXPERIMENTS

In this paper the performance of the Contourlet transform is been evaluated. In the first experiment template matching of a noisy image using Contourlet based blur invariants is performed. The image registration is performed perfectly using the Contourlet transform without any problem. In the second experiment the image quality of the template images in Contourlet transform is being evaluated using the PSNR values and the Wavelet transform is being used for comparison.

#### A. Image Registration

In this experiment we have used "9-7" biorthogonal filters for the wavelet transform with 6 decomposition levels. For the contourlet transform, in the LP stage we also use the "9-7" filters. Three template images are taken from the images Cameraman, Barbara and Goldhill. These template images are then degraded using Gaussian noise as shown in figure 4. These degraded template images are then matched with the original image. The size of the images is 512×512, and the template is 218×218. For this experiment moments of order up to 7 are calculated and totally 18 invariants are used for performing the template matching. For registration, a window of the same size of the degraded template is chosen, and the corresponding moments of that section of the image are calculated. This window is moved all around the image until it scans it thoroughly Afterwards, the following measure is used to calculate the similarity of the template to every section of the image:

section of the image:
$$S_{ij} = exp\left(-\sum_{n=1}^{N} \left| \frac{B^{y}(n) - B^{x}(n)}{B^{x}(n)} \right| \right)$$

B is the array of calculated invariants, x is the template, and y is a section of the target photo. From figure 5 we can say that the images were perfectly registered using the Contourlet based blur invariants.



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Fig- 4 Degraded Template Image

#### B. Experiment II

In this experiment the image quality of the template image after using Contourlet transform is being evaluated. The template images are degraded using Gaussian noise and salt & pepper noise. The Peak Signal to Noise Ratio (PSNR) values of these images is calculated in order to determine the image quality. The calculated PSNR usually adopts a dB value for quality judgment, the larger PSNR is, the higher the image quality (which means there is a little difference between the original image and Blur image). On the contrary smaller dB value means there is a more distortion. The PSNR is defined as:

$$PSNR = 10 \log \left( \frac{255^2}{MSE} \right)$$

where, MSE=Mean Square Error value which is given by

$$MSE = \left(\frac{1}{M \times N}\right) \sum_{i=1}^{M} \sum_{j=1}^{N} (X_{ij} - Y_{ij})^2$$

where,

 $M \times N =$ Size of the Blur Image,

 $X_{ij}, Y_{ij}$  = pixel values of the original image and Blur image respectively

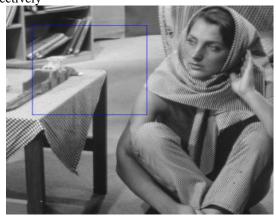
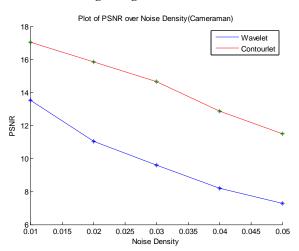






Fig- 5 Registration results





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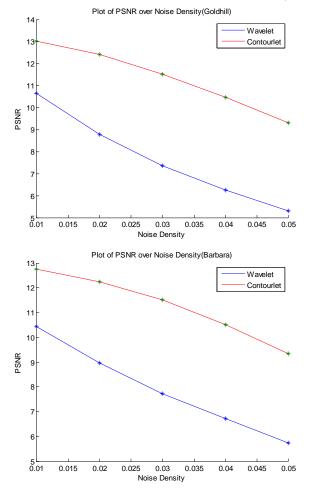
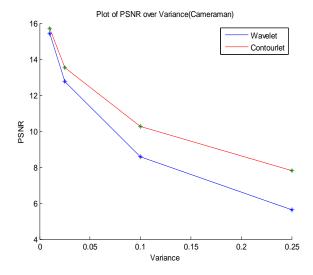
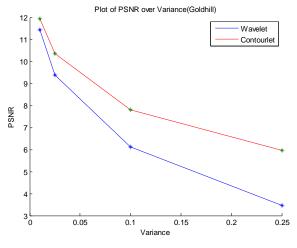


Fig- 6 Comparison Graph of PSNR (dB) for Contourlet and Waveletusing Salt &Pepper noise

Figure 6 and Figure 7 show the comparison of the Contourlet and Wavelet over the PSNR values. We can see that as the noise keeps on increasing the PSNR values decrease but the Contourlet shows better performance than the Wavelet.





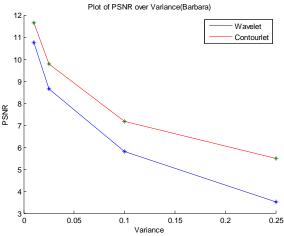


Fig-7Comparison Graph of PSNR (dB) for Contourlet and Waveletusing Gaussian noise

#### V. CONCLUSIONS

In this paper we have used Contourlet transform along with blur invariants to evaluate the performance of the Contourlet transform. The contourlet transform is a discrete extension of the curvelet transform that aims to capture curves instead of points, and provides for directionality and anisotropy. The main differences between othersystems and our Contourlet construction are that the previousmethods do not allow for a different number of directionsat each scale while achieving nearly critical sampling. Inaddition, our construction employs iterated filter banks, whichmakes it computationally and there is a preciseconnection with continuous-domain expansions. Two experiments have been carried out to evaluate its performance In the first experiment image registration has been carried out using the Contourlet transform. Three template images were chosen and degraded using Gaussian noise. Inspite of the presence of severe degradation the images were registered perfectly in Contourlet transform using blur invariants The second experiment the performance of the Contourlet was compared with the Wavelet using the PSNR values. The images were degraded using Gaussian noise and salt & pepper noise. The image quality of the degraded images was better in the Contourlet compared to the Wavelet There have been various



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researches being carried out on the Contourlet transform. Evaluating the performance of the Contourlet transform for various image processing techniques by using different Contourlet filters is the future research for the proposed system.

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